Model-theoretic Semantics

Semantics Intro I

- What is the semantics of the following statement, according to RDF and according to RDFS?

```
Ex:SpaceAccessory rdfs:subClassOf ex:Product
```

Semantics Intro II

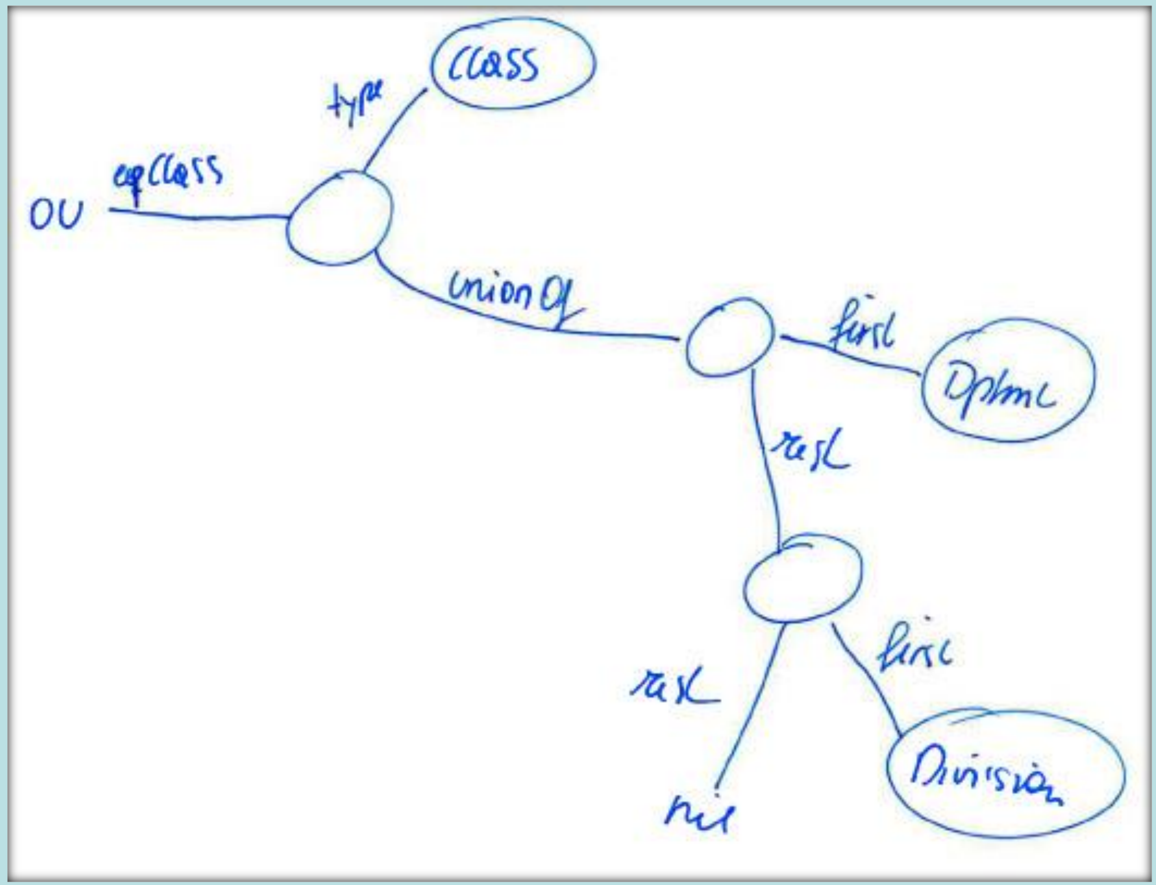
What is the semantics of the following ontology, according to RDF(S) and according to OWL2?


```
<rdf:RDF
xmlns="http://www.semanticweb.org/ontologies/2012/10/Ontology1352
    xml:base="http://www.semanticweb.org/ontologies/2012/10/Ontolog
    xmlns:rdfs="http://www.w3.org/2000/01/rdf-schema#"
    xmlns:owl="http://www.w3.org/2002/07/owl#"
    xmlns:xsd="http://www.w3.org/2001/XMLSchema#"
    xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"

xmlns:Ontology1352809960588="http://www.semanticweb.org/ontologi
809960588.ow
   <owl:Ontolo
rdf:about="http
      <rdfs:labe
      <owl:vers
      <owl:prior
      <owl:impc
   </owl:Ontolc

   <owl:Class r
      <owl:equi
         <owl:C
            <owl
               <rdf:Description rdf:about="&Ontology1352809960588;De
               <rdf:Description rdf:about="&Ontology1352809960588;Div
            </owl:unionOf>
         </owl:Class>
      </owl:equivalentClass>
      <rdfs:subClassOf rdf:resource="&owl;Thing"/>
   </owl:Class>

</rdf:RDF>
```



```
<http://www.semanticweb.org/ontologies/2012/10/&On
> <http://www.w3.org/2002/07/owl#equivalentClass> _
_:bnode1447883008 <http://www.w3.org/1999/02/22-r
<http://www.w3.org/2002/07/owl#Class>.
_:bnode1447883008 <http://www.w3.org/2002/07/owl#
_:bnode410442880 <http://www.w3.org/1999/02/22-rd
<http://www.semanticweb.org/ontologies/2012/10/&On
_:bnode410442880 <http://www.w3.org/1999/02/22-rd
_:bnode2037950656 <http://www.w3.org/1999/02/22-r
<http://www.semanticweb.org/ontologies/2012/10/&On
_:bnode2037950656 <http://www.w3.org/1999/02/22-r
<http://www.w3.org/1999/02/22-rdf-syntax-ns#nil>.
```


http://www.rdfabout.com/demo/validator/



Model-Theoretic Semantics

So far we have been talking quite informally about semantics…

What follows is really a crash-course in model-theoretic semantics plus how to use these to assign meaning to RDFS and OWL 2 language constructs that glosses over many details. To know more, read:

http://www.w3.org/TR/rdf-mt/ for RDF Semantics

http://www.w3.org/TR/owl2-direct-semantics/ for OWL2 direct semantics

Logic Theories, Interpretations and Models

Very roughly:

- Logical statements (axioms)
- Logical theory: A collection of logical statements (a knowledge base, an ontology)
- Many interpretations („worlds“)

- Some interpretations satisfy the conditions of a theory, these are models of the theory.
- Some theories are satisfied by no interpretation (unsatisfiable)

Interpretations

Interpretation I = (D, $.^I$) for a vocabulary V:

- Domain of discourse D
- Function $.^I$ that maps individuals in V to elements in D, unary predicates (classes) in V to subsets of D and binary predicates (properties) to elements in (D x D).

Interpretation Example – Text (please use text notation in exams!!!)

V = {Mackenzie, NBDD, Deparmtent, manages}

D = {Lisa, Anna, Mary, x, y, z}

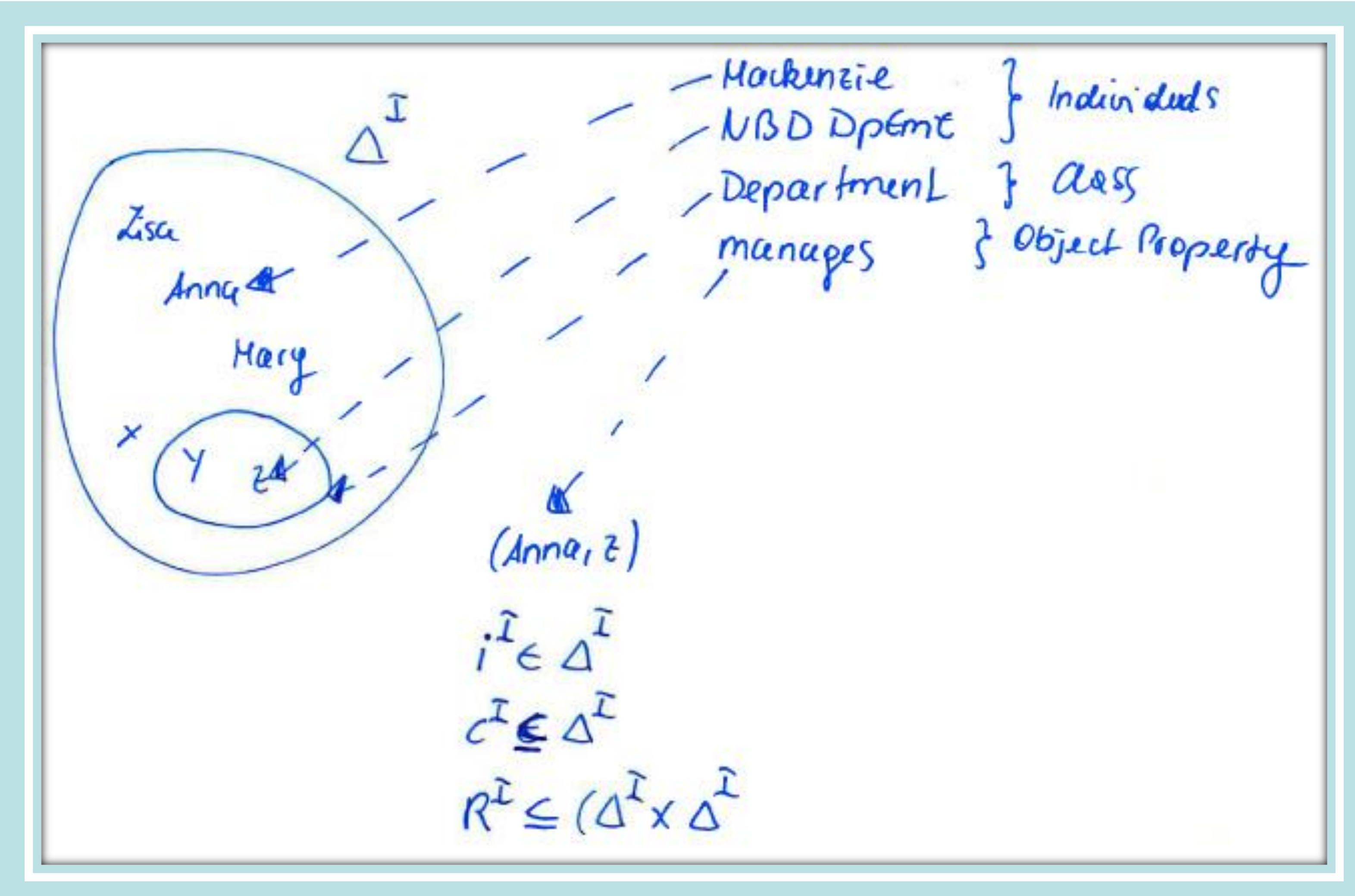
Mackenzie^I = Anna

NBDD^I = z

Department^I = {y, z}

Manages^I = {(Anna,z)}

Interpretation Example - Graphical



Models

An interpretation relates a domain of discourse (D) to a vocabulary (V).

The vocabulary is used to express a logic theory (one way of seeing a knowledge base or ontology)

<u>Theory:</u>
ClassAssertion(:Department :NBDDptmt)

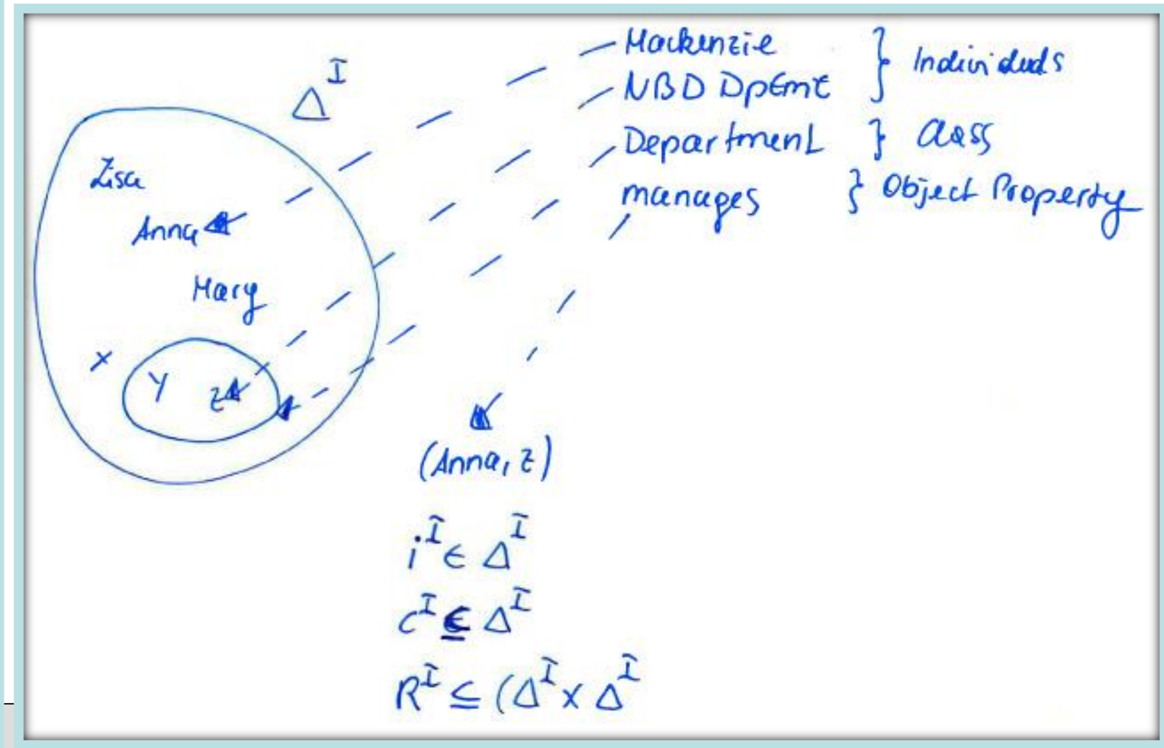
An interpretation I is a model of a theory O (for ontology) iff I satisfies O.

Example

Is the theory below satisfied by the interpretation below?

What do we need in order to answer this question?

Theory:
ClassAssertion(:Department :NBDDptmt)



OWL 2 Semantics

Axiom	Condition
SubClassOf(CE_1 CE_2)	$(CE_1)^C \subseteq (CE_2)^C$
EquivalentClasses(CE_1 ... CE_n)	$(CE_j)^C = (CE_k)^C$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$
DisjointClasses(CE_1 ... CE_n)	$(CE_j)^C \cap (CE_k)^C = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$
DisjointUnion(C CE_1 ... CE_n)	$(C)^C = (CE_1)^C \cup ... \cup (CE_n)^C$ and $(CE_j)^C \cap (CE_k)^C = \emptyset$ for each $1 \leq j \leq n$ and each $1 \leq k \leq n$ such that $j \neq k$

Axiom	Condition
ClassAssertion(CE a)	$(a)^I \in (CE)^C$
ObjectPropertyAssertion(OPE a_1 a_2)	$((a_1)^I , (a_2)^I) \in (OPE)^{OP}$

http://www.w3.org/TR/owl2-semantics/

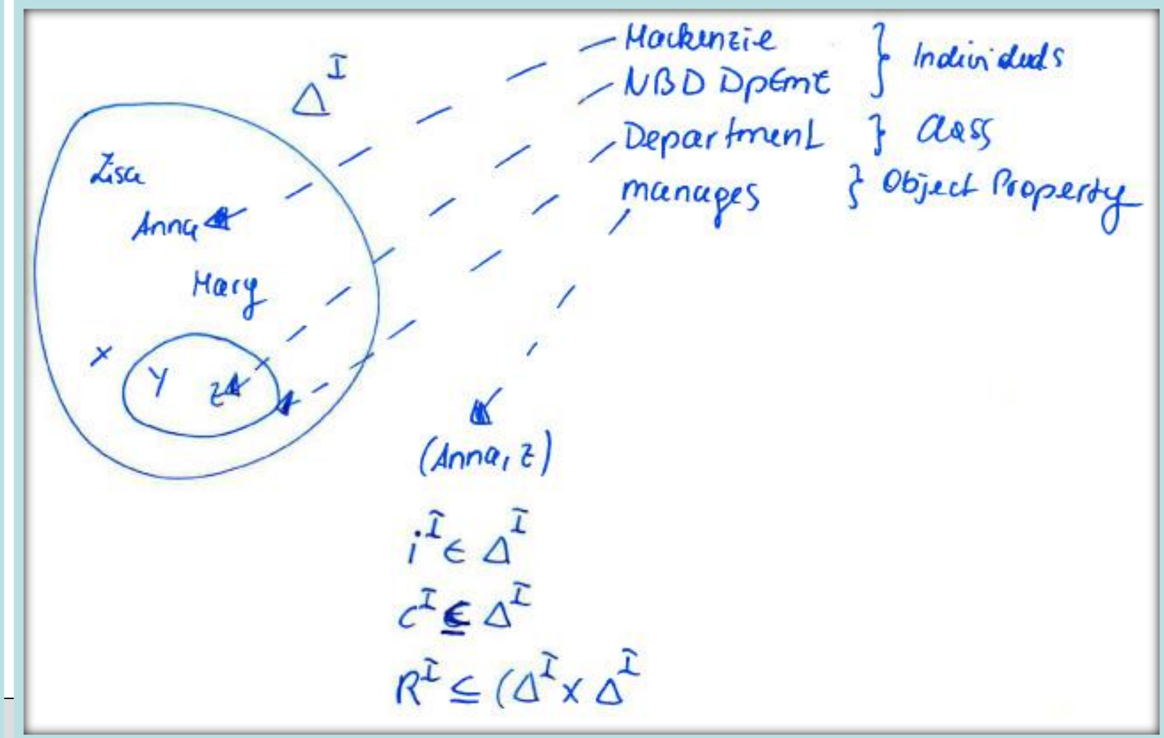
Description	Interpretation
owl:Thing	Δ_I
owl:Nothing	empty set
ObjectComplementOf(C)	$\Delta_I \setminus C^{Ic}$
ObjectIntersectionOf(C_1 ... C_n)	$C_1^{Ic} \cap ... \cap C_n^{Ic}$
ObjectUnionOf(C_1 ... C_n)	$C_1^{Ic} \cup ... \cup C_n^{Ic}$
ObjectOneOf(a_1 ... a_n)	$\{ a_1^{Ii} , ... , a_n^{Ii} \}$
ObjectSomeValuesFrom(R C)	$\{ x \mid \exists y : (x, y) \in R^{Ipo}$ and $y \in C^{Ic} \}$
ObjectAllValuesFrom(R C)	$\{ x \mid \forall y : (x, y) \in R^{Ipo}$ implies $y \in C^{Ic} \}$
ObjectHasValue(R a)	$\{ x \mid (x, a^{Ii}) \in R^{Ipo} \}$
ObjectExistsSelf(R)	$\{ x \mid (x, x) \in R^{Ipo} \}$
ObjectMinCardinality(n R C)	$\{ x \mid \#\{ y \mid (x, y) \in R^{Ipo}$ and $y \in C^{Ic} \} \geq n \}$
ObjectMaxCardinality(n R C)	$\{ x \mid \#\{ y \mid (x, y) \in R^{Ipo}$ and $y \in C^{Ic} \} \leq n \}$
ObjectExactCardinality(n R C)	$\{ x \mid \#\{ y \mid (x, y) \in R^{Ipo}$ and $y \in C^{Ic} \} = n \}$
DataSomeValuesFrom(U_1 ... U_n DR)	$\{ x \mid \exists y_1, ..., y_n : (x, y_k) \in U_k^{Ipd}$ for each $1 \leq k \leq n$ and $(y_1, ..., y_n) \in DR^D \}$
DataAllValuesFrom(U_1 ... U_n DR)	$\{ x \mid \forall y_1, ..., y_n : (x, y_k) \in U_k^{Ipd}$ for each $1 \leq k \leq n$ implies $(y_1, ..., y_n) \in DR^D \}$
DataHasValue(U v)	$\{ x \mid (x, v^D) \in U^{Ipd} \}$
DataMinCardinality(n U DR)	$\{ x \mid \#\{ y \mid (x, y) \in U^{Ipd}$ and $y \in DR^D \} \geq n \}$
DataMaxCardinality(n U DR)	$\{ x \mid \#\{ y \mid (x, y) \in U^{Ipd}$ and $y \in DR^D \} \leq n \}$
DataExactCardinality(n U DR)	$\{ x \mid \#\{ y \mid (x, y) \in U^{Ipd}$ and $y \in DR^D \} = n \}$

Example

Same example, plus we know:

ClassAssertion(C a) is satisfied by I iff $(a)^I \in (C)^I$

Theory:
ClassAssertion(:Department :NBDDptmt)



Reasoning (via Example)

Given that SubClassOf(CE_1 CE_2) is satisfied by I iff $(CE_1)^I \subseteq (CE_2)^I$:

How could you find out whether:

- ex:TextBook rdfs:SubClassOf ex:Publication follows from these 2 triples?

  ex:TextBook rdfs:subClassOf ex:Book

  ex:Book rdfs:subClassOf ex:Publication

- The following triple is true?

  ex:TextBook rdfs:subClassOf ex:TextBook

OWL 2 Semantics

… correspond largely to the semantics of the DL SROIQ

... SROIQ however misses datatypes and datatype properties, and metamodeling

We have been talking about the direct model semantics of OWL2.

However, rdf-based semantics for OWL2 also exist, and the semantics can be different. To read more, consult:

- Domingue, Fensel, Hendler: Handbook of Semantic Web Technologies, p.384ff
- http://www.w3.org/TR/owl2-rdf-based-semantics/#Example_on_Semantic_Differences

Why should we care about semantics?

Gives you a basic understanding about interpretations, models, logic theories and how to find out what a logic theory (=any data or knowledge model in RDFS or OWL2) means.

Lets you understand what reasoners do when they:

- Check the satisfiability of an ontology O: Is there a model of O?
- Check the satisfiability of a concept C w.r.t. an ontology O: Is there a model of O in which C^I is non-empty?
- Does the ontology O entail (=implicitly state) that a is of type C: Is $a^I \in C^I$ in all models of O?

Why should we care about semantics?

Lets you understand why a simple query language can only find out about explicitly stated knowledge, and you need a reasoner to „elicit“ the implicit knowledge.

Exercise 1

Our ontology O consists of two axioms:

- ClassAssertion(ex:Manager ex:Mackenzie)
- ObjectPropertyAssertion(ex:manages ex:Mackenzie ex:NBDD)

a) Write down an interpretation that satisfies O (is a model of O).

b) Write down an interpretation that does not satisfy O.

Exercise 2

An ontology O consists of the following axioms:

- SubClassOf(ex:SpaceSuit ex:Product)
- SubClassOf(ex:SpaceAccessory ex:Product)

Is the following interpretation I a model of O or not?

D = {a,b,c,d,e}

$SpaceSuite^I$ = {a,b,c}

$SpaceAccessory^I$ = {b,c,d}

$Product^I$ = {a,c,d}

If not, how could you change I so that it satisfies O?